Math 112: Business Calculus
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Essential Course Info

My Course Website:	math.washington.edu/~aloveles/
Homework Log-In (use UWNetID):	webassign.net/washington/login.html

First week to do list

1. Read 9.3 and 9.4 of the book. Start attempting HW.
2. Print off the **Activity** and bring it to quiz section.

Today

- Syllabus/Intro
- Section 9.3: Rates!

1st HW assignments

Closing time is always 11pm.

- HW 9.3 closes Tuesday
- HW 9.4 closes Thursday

What we will do in this course:
We learn basic concepts, tools and terminology from calculus with applications to business/economics.

1. **Ch. 9 – Derivatives**
2. **Ch. 10 – Derivative Applications**
   - Business functions
   - Graphing, Max/Min
3. **Ch. 11 – More Deriv. Rules**

4. **Ch. 12 – Basics of Integrals**
5. **Ch. 13 – Integral Applications**
   - Area, antiderivatives
   - Business functions
   - Supply/Demand

6. **Ch. 14 – Multivariable Calculus**

7. **Practicing Algebra and Precalc**

Students often say: The hardest part of calculus is you have to know all your precalculus, and they are right.

Improving your algebra and precalculus skills will be one of the best benefits you will gain from this course. (Arguably as valuable as the course content itself).

9.3: Rate of Change

Example: Ron and Harry go for a drive. Here is the total distance they have traveled at various times:

t (hours)	0	0.5	1.0	1.5	2.0
$D(t)$ (miles)	0	8	15	22	50

The ***rate of change*** of distance is the ratio (fraction) of change in distance over time. Briefly,

$$Rate = \frac{\Delta Dist}{\Delta Time}$$

We use the word "speed" when talking about "rate of change of distance"

Q: What is the (overall) average speed from t = 0 to t = 2 hours?

Q: What is the average speed from t = 1.5 to t = 2 hrs?

Q: What is the speedometer speed at $t = 2$ hrs?

A: We don't have enough information to say for sure, we can only estimate.

We call this the ***instantaneous rate of change of distance at t = 2*****.**

(or just the rate of change **at** $t = 2$).

Math 112 is all about instantaneous rates of change and the many things we know about them. We will:

1. Develop tools to quickly find rates at a point.
2. Use these tools to go between our business functions.
3. Use these tools to analyze our business functions (max/min, increasing/decreasing, and more).
4. Learn the language of rates and calculus which you need in business and economics.

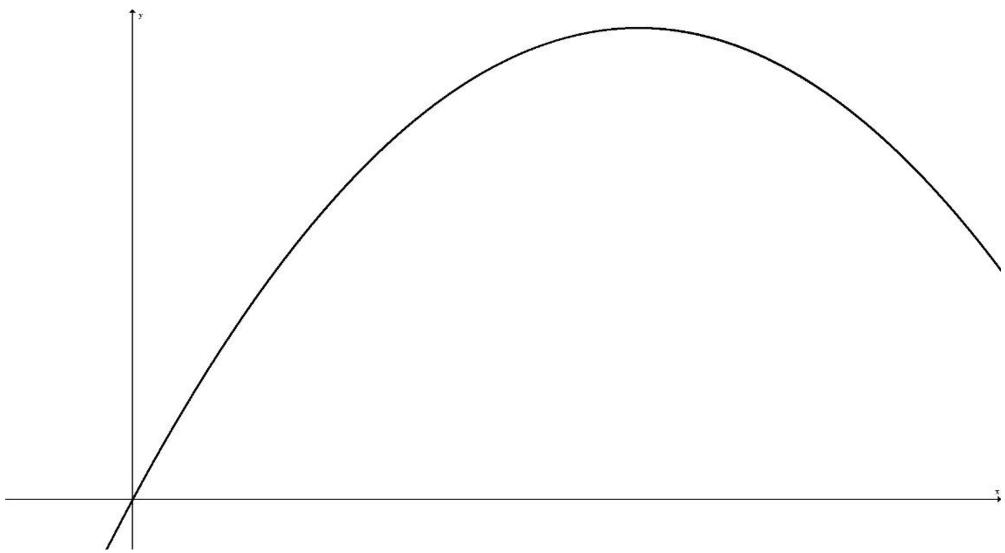
Now assume we have an algebraic rule instead of a table:

Example: Assume Tommy is in a train. His distance from the starting line (in feet) after t seconds is given by

$$D(t) = 128t - 16t^2$$

Q: What is the average speed from $t = 3$ to $t = 4$ seconds?

How would you do the last question graphically?



NOTE/Review:

A *secant* line goes through a graph at two points. A *tangent* line just touches a graph at one point with the same slope as the graph at that point.

slope of secant = average rate

$$= \frac{f(b) - f(a)}{b - a}$$

slope of tangent = instantaneous rate

$$= f'(a)$$

Q: What is the instantaneous speed at t = 3 seconds?

We give this the notation:

$D'(3)$ = "instantaneous speed at t=3"

We don't know the tools to find this exactly yet, but, we can approximate:

Idea: Let's find the average speed from t = 3 to t = 3.01 seconds and use that as an approximation.

Example: $f(x) = x^2 - 4x + 5$

Let's try to compute $f'(3)$.

Idea: Use a second point nearby

slope from 3 to 3.1	$\frac{f(3+0.1)-f(3)}{3.1-3} = \frac{[(3.1)^2 - 4(3.1)+5] - [(3)^2 - 4(3)+5]}{3.1-3}$ $= \frac{2.21-2}{0.1} = \frac{0.21}{0.1} = 2.1$
slope from 3 to 3.01	$\frac{f(3+0.01)-f(3)}{3.01-3} = \frac{[(3.01)^2 - 4(3.01)+5] - [(3)^2 - 4(3)+5]}{3.01-3}$ $= \frac{2.0201-2}{0.01} = \frac{0.0201}{0.01} = 2.01$
slope from 3 to 3.001	$\frac{f(3+0.001)-f(3)}{3.001-3} = \cdots = 2.001$
slope from 3 to 3.0001	$\frac{f(3+0.0001)-f(3)}{3.0001-3} = \cdots = 2.0001$

It appears the secant slope is getting closer and closer to 2 as the second point gets closer. Now let's do this systematically with algebra:

First, a key shortcut: Instead of adding 0.1 or 0.01 or 0.001, let's label this amount by a symbol: *h*

For each approx., we were computing

$$f'(3) \approx \frac{f(3+h) - f(3)}{(3+h) - 3} = ??$$

It becomes very easy to see the final answer if we can expand and simplify with algebra ***before*** plugging in numbers. Let's try it:

Recall: $f(x) = x^2 - 4x + 5$

What is $f'(3)$?

Expand and completely simplify

$$\frac{f(3+h) - f(3)}{(3+h) - 3}$$

Same function: $f(x) = x^2 - 4x + 5$ What is $f'(5)$?

Find $f'(5)$ by using the same process.

Expand and completely simplify

$$\frac{f(5+h) - f(5)}{(5+h) - 5}$$

It is better just to do this once!

Same function: $f(x) = x^2 - 4x + 5$

Find $f'(a)$ by using the same process.

$$\frac{f(a+h) - f(a)}{(a+h) - a}$$

What is $f'(a)$?

Check: Does this match f'(3) and f'(5)?